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Generalized screening theorem for Higgs decay processes in the two-doublet model

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Abstract

The radiative corrections to the decay processes of the neutral (CP-even) Higgs boson (H) into a longitudinal gauge boson pair, *i.e.*, $H \to Z_L Z_L$ and $H \to W_L^+ W_L^-$ are analyzed in the two-Higgs doublet model by assuming that all of the Higgs boson masses are much greater than the W and Z bosons'. These calculations are motivated to see if one could see potentially large virtual effects to these decay rates due to the charged and CP-odd neutral Higgs boson masses (m_G and m_A, respectively) which are supposed to be larger than m_H. It is pointed out that, although the radiative corrections to the decay width $\Gamma(H \to W_L^+ W_L^-)$ depend sensitively in general on m_G and m_A, there occurs a screening effect, *i.e.,* cancellation in leading terms once we set $m_G = m_A$, so that the radiative corrections tend to be minimized. It is also pointed out that the decay rate $\Gamma(H \to Z_L Z_L)$ is fairly insensitive to the other heavier Higgs masses and is possibly a good measuring tool of the Higgs mixing angle. The mechanism of these screening phenomena in the Higgs decays is explained on the basis of a new screening theorem, which we postulate with reference to the custodial symmetry in the Higgs potential.

1. Introduction

It is becoming more and more important to investigate non-decoupling effects of heavy particles in the low energy observables, since the available energy of future accelerators will increase only little by little. One of the examples of low energy manifestations of heavy particles has been given by the top quark, whose mass was surmised before its discovery [1,2] by detailed analyses of the electroweak data.

Now that the existence of the top quark has been confirmed, the next pressing experimental task is the discovery of the Higgs boson. (For a review, see Ref. [3].) It has been known for long time that the Higgs boson in the standard model is very elusive: indirect signatures of the Higgs boson appear in the low energy data on the oblique-type radiative corrections at most in logarithmic terms at the one loop level. This fact is often referred to as Veltman's screening theorem [4]. Perhaps more precise electroweak data would change the situation and the signatures of the standard model Higgs boson might be just around the corner. In any case, once the existence of the standard model Higgs boson would be established, we should be still looking after new unknown heavy particles even further, via radiative corrections in order to probe what lies beyond the standard model.

In our previous publication [5], we have investigated the possibility that one could get hold of signatures of unknown heavy particles through radiative corrections, supposing that the standard model Higgs boson H has been discovered. As an example we adopted a two-Higgs doublet model, the most conservative extension of the standard model. Here the Higgs sectors consist of, besides the neutral H boson, another (CP-even) neutral Higgs boson h, (CP-odd) neutral Higgs boson A, and the charged Higgs boson $G^{\pm}$. Assuming that their masses m_H, m_h, m_A and m_G are all much greater than M_W, we computed the radiative corrections to the decay width $\Gamma(H \to W_L^+ W_L^-)$ as a function of m_h, m_A, and m_G. Here the subscript L denotes the longitudinal polarization . The preliminary numerical calculations in

Ref. [5] show that the magnitude of radiative corrections depends rather sensitively on the choice of m_G and m_A. It is also suggested implicitly that, if $m_G = m_A$, then the radiative corrections are minimized. Note in this connection that the Higgs potential in the two-doublet model respects an $SU(2)_L \times SU(2)_R$ symmetry [6], if we put $m_G = m_A$. The existence of the additional global $SU(2)_R$ symmetry would lead to isospin symmetry $SU(2)_V = \mathrm{diag}[SU(2)_L \times SU(2)_R]$ after spontaneous symmetry breaking. This isospin symmetry is often called custodial symmetry in literatures.

The purpose of the present paper is two-fold. The first one is to extend our previous analyses to another decay mode $H \to Z_L Z_L$ and to compare its dependences on m_G, m_A, m_h and Higgs mixing angles (α and β) with those in $H \to W_L^+ W_L^-$. We would like to establish the following two peculiar facts as to the radiative corrections by numerical calculations on computer.

(1) Although the radiative corrections to the decay $H \to W_L^+ W_L^-$ is in general sensitive to the choice of m_G and m_A, they tend to be minimized if the mass parameters are such that the custodial symmetry is respected, *i.e.*, $m_G = m_A$.

(2) The radiative corrections to the decay $H \to Z_L Z_L$ are insensitive to the choice of the mass parameters. It is only sensitive to the Higgs mixing angles.

The second purpose of the present work is to give theoretical explanations to (1) and (2) without recourse to numerical methods. We will postulate a new screening "theorem" for the Higgs decay vertices, by which we are able to acquire a clear grasp of m_G-, m_A- and m_h-dependences of the decay widths $\Gamma(H \to W_L^+ W_L^-)$ and $\Gamma(H \to Z_L Z_L)$.

The screening theorem for the Higgs vertices that we just mentioned is in close analogy with the celebrated Veltman's theorem [4], which may be applied, in contrast, to the oblique-type radiative corrections. The statement in (1) referring to the custodial symmetry reminds us of the detailed study of the Veltman's theorem

in the standard model by Einhorn and Wudka [7]. They argued that the radiative corrections to the gauge boson propagators may be classified into several types. On the basis of the custodial symmetry in the weak $U(1)_Y$-coupling limit, they claimed that the Higgs mass dependence at the L-th loop is at most $(m_H^2)^{L-1}$ rather than $(m_H^2)^L$. The statement (1) indicates that there is a similar situation in the two-Higgs doublet model as well, in radiative corrections to the (non-oblique type) decay process if the custodial symmetry is respected. We will make this fact crystalline in the form of a new screening theorem. Although we will not go so far as to give as general an argument as Einhorn and Wudka's including all order corrections, the similarity lying between Veltman's theorem and our counterpart provides a strong evidence in favor of the validity of our version beyond the one-loop level.

Throughout the present paper we will often refer to and make use of some of the formulae given in Ref. [5] with respect to the decay $H \to W_L^+ W_L^-$. In Ref. [5], in passing, the effect of the top quark mass was not taken into consideration. Since the top quark mass might produce non-negligible effects to the decay [8], we will improve our previous analysis by including quark loops as well. Partial lists of earlier works on these decay processes in the standard model and supersymmetric models are given in Ref. [9-12]. A brief summary of the present work is found in Ref. [13].

The present paper is organized as follows. Our Higgs potential is specified in Sec. 2, thereby explaining the connection between the custodial symmetry and the mass degeneracy $m_G = m_A$. We will make an extensive use of the equivalence theorem [14-17] at the loop level. Sec. 3 is devoted to clarification of an issue that arises if one uses the equivalence theorem at higher orders [17]. One loop calculations of the decay processes, which will be presented in Sec. 5, are rendered complicated, because of the mixing between those of same quantum numbers. The method of renormalization of the Higgs mixing angles and the wave function renormalization

constant matrices are explained at length in Sec. 4. Our numerical calculation is presented in Sec. 6. Our new screening theorem is given in Sec. 7 which, we believe, will lay cornerstones for clear understanding of the qualitative features of the numerical calculations in Sec. 6. Sec. 8 is devoted to summary and discussions. Some details of our calculations are relegated to several Appendices.

2. The Higgs potential and the custodial symmetry

Before launching into the details of our calculations, we have to specify our Higgs potential of the two Higgs doublets, Φ_1 and Φ_2 with $Y = 1$. The criterion of determining the Higgs potential is the natural suppression of the flavor-changing neutral current [18] and it is often assumed that the discrete symmetry $\Phi_2 \to -\Phi_2$ is respected except for soft terms, namely,

$$\begin{aligned} V(\Phi_1, \Phi_2) &= -\mu_1^2 \mid \Phi_1 \mid^2 - \mu_2^2 \mid \Phi_2 \mid^2 - (\mu_{12}^2 \Phi_1^\dagger \Phi_2 + \mu_{12}^{2*} \Phi_2^\dagger \Phi_1) \\ &\quad + \lambda_1 \mid \Phi_1 \mid^4 + \lambda_2 \mid \Phi_2 \mid^4 + \lambda_3 \mid \Phi_1 \mid^2 \mid \Phi_2 \mid^2 \\ &\quad + \lambda_4 (\mathrm{Re}\Phi_1^\dagger \Phi_2)^2 + \lambda_5 (\mathrm{Im}\Phi_1^\dagger \Phi_2)^2. \end{aligned} \tag{1}$$

This potential is general enough to encompass supersymmetric models and the soft term $(\mu_{12}^2 \Phi_1^\dagger \Phi_2 + \mu_{12}^{2*} \Phi_2^\dagger \Phi_1)$ plays an important role there. The complex phase of μ_{12} may also be instrumental to the idea of baryogenesis at the electro-weak scale [19]. The existence of this term, however, makes our analysis too complicated to get an insight into radiative corrections. We will therefore set $\mu_{12} = 0$ throughout our calculations. Perhaps it is worth mentioning as another excuse for setting $\mu_{12} = 0$ that, since we are interested in the non-decoupling effects caused by strong quartic couplings (times $v^2 \approx (246\mathrm{GeV})^2$), the mass scale μ_{12} as opposed to v does not have direct relevance to what we are concerned with and may be neglected at the first step.

From the viewpoint of the custodial symmetry, it is convenient to introduce a 2×2 matrix notation of the Higgs fields, *i.e.*, $\mathbf{\Phi}_i = (i\tau_2\Phi_i^*, \Phi_i)$. In terms of this notation, our Higgs potential is expressed as

$$\begin{aligned} V(\Phi_1,\Phi_2) \quad = \quad & -\frac{1}{2}\mu_1^2 \mathrm{tr}(\mathbf{\Phi}_1^\dagger \mathbf{\Phi}_1) - \frac{1}{2}\mu_2^2 \mathrm{tr}(\mathbf{\Phi}_2^\dagger \mathbf{\Phi}_2) \\ & +\frac{1}{4}\lambda_1 \left\{\mathrm{tr}(\mathbf{\Phi}_1^\dagger \mathbf{\Phi}_1)\right\}^2 + \frac{1}{4}\lambda_2 \left\{\mathrm{tr}(\mathbf{\Phi}_2^\dagger \mathbf{\Phi}_2)\right\}^2 \\ & +\frac{1}{4}\lambda_3 \mathrm{tr}(\mathbf{\Phi}_1^\dagger \mathbf{\Phi}_1)\mathrm{tr}(\mathbf{\Phi}_2^\dagger \mathbf{\Phi}_2) + \frac{1}{16}\lambda_4 \left\{\mathrm{tr}(\mathbf{\Phi}_1^\dagger \mathbf{\Phi}_2) + \mathrm{tr}(\mathbf{\Phi}_2^\dagger \mathbf{\Phi}_1)\right\}^2 \\ & -\frac{1}{16}\lambda_5 \left\{\mathrm{tr}(\tau_3 \mathbf{\Phi}_2^\dagger \mathbf{\Phi}_1) - \mathrm{tr}(\tau_3 \mathbf{\Phi}_1^\dagger \mathbf{\Phi}_2)\right\}^2 . \end{aligned} \tag{2}$$

This shows clearly that, without the last term in Eq. (1), the potential would possess the global symmetry $SU(2)_L \times SU(2)_R$, under which $\mathbf{\Phi}_i$ undergoes the transformation $\mathbf{\Phi}_i \to g_L \mathbf{\Phi}_i g_R$ ($g_L \in SU(2)_L$, $g_R \in SU(2)_R$). The isospin symmetry $SU(2)_V = \mathrm{diag}[SU(2)_L \times SU(2)_R]$, that would survive the spontaneous symmetry breaking, is also broken by the λ_5-interactions exclusively. We will come to this point later in connection with the screening theorem.

The particle content of the scalar sector may be seen by putting

$$\Phi_i = \begin{pmatrix} w_i^+ \\ \frac{1}{\sqrt{2}}(v_i + h_i + iz_i) \end{pmatrix} \tag{3}$$

and by shuffling Eq. (1). Here v_i's (i=1, 2) are the vacuum expectation values. The mass eigenstates are obtained by diagonalizing the quadratic terms in the neutral as well as charged sectors via

$$\begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} h \\ H \end{pmatrix}, \tag{4}$$

$$\begin{pmatrix} w_1 \\ w_2 \end{pmatrix} = \begin{pmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} w \\ G \end{pmatrix}, \tag{5}$$

$$\begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = \begin{pmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} z \\ A \end{pmatrix}. \tag{6}$$

The mixing angle β is simply given by $\tan\beta = v_2/v_1$. As we can see easily, H and h are CP-even neutral field, while A a CP-odd neutral one. The charged Higgs field is denoted by $G^\pm$, and the Nambu-Goldstone bosons are $w^\pm$ and z.

The physical parameters of our theory are the masses m_H, m_h, m_G, m_A and the vacuum expectation value $v = \sqrt{v_1^2 + v_2^2}$ together with the mixing angles α and β. The quartic couplings in (1) are expressed in terms of these physical parameters by

$$\lambda_1 = \frac{1}{2v^2\cos^2\beta}(m_h^2\cos^2\alpha + m_H^2\sin^2\alpha), \tag{7}$$

$$\lambda_2 = \frac{1}{2v^2\sin^2\beta}(m_h^2\sin^2\alpha + m_H^2\cos^2\alpha), \tag{8}$$

$$\lambda_3 = \frac{\sin 2\alpha}{v^2\sin 2\beta}(m_h^2 - m_H^2) + \frac{2m_G^2}{v^2}, \tag{9}$$

$$\lambda_4 = -\frac{2m_G^2}{v^2}, \tag{10}$$

$$\lambda_5 = \frac{2}{v^2}(m_A^2 - m_G^2). \tag{11}$$

As wee see in Eq. (11), the deviation from the mass degeneracy $m_G^2 = m_A^2$ between charged and *CP*-odd neutral Higgs bosons thus measures the custodial symmetry breaking.

The quartic couplings are all assumed to be real and there is no source of CP-violation in the Higgs potential. The CP-invariance enables us to set up several selection rules for triple and quartic Higgs couplings. It is worthwhile mentioning that we could assign G-parity in the usual way as in hadron physics in connection with the isospin symmetry. The neutral h and H bosons have even G-parity, while G, A, w and z odd G-parity. This G-parity is also useful to set up selection rules for the Higgs-Goldstone interactions for the case of $\lambda_5 = 0$.

3. The equivalence theorem at the loop level

The Higgs boson decay into a gauge boson pair is dominated preferentially by those into longitudinally polarized ones if $m_H \gg M_Z, M_W$. In such a case we may take an advantage of the equivalence theorem [14-17]. This theorem states that the S-matrix elements associated with longitudinal gauge bosons are approximated by those of corresponding Nambu-Goldstone bosons;

$$\begin{aligned} & T(Z_L(p_1), \cdots, Z_L(p_n), W_L(q_1), \cdots, W_L(q_m)) \\ = \; & (C^Z_{\rm mod})^n (C^W_{\rm mod})^m T(iz_(p_1), \cdots, iz_(p_n), iw(q_1), \cdots, iw_(q_m)) + \mathcal{O}(M_W/\sqrt{s}). \end{aligned} \tag{12}$$

Here $\sqrt{s}$ is the typical energy scale characterizing the scattering process and $C^Z_{\rm mod}$, and $C^W_{\rm mod}$ are the so-called modification factor [17] to be attached to each external gauge boson line of Z and W's, respectively.

The equivalence theorem was first proved on the tree level [14, 15]. Since then, the validity of this theorem on the loop level has been examined by several authors [16-17]. The point is that the right-hand side of Eq. (12) is unphysical (gauge-dependent) matrix elements and that we have to specify how to renormalize the external lines of the Nambu-Goldstone bosons. The modification factors $C^Z_{\rm mod}$ and $C^W_{\rm mod}$ are thus introduced to match the external line renormalization to the physical S-matrix on the left-hand side. In a nutshell, if we work in the on-shell renormalization scheme in the Landau gauge, these modification factors turn out at the one-loop level to be [17]

$$C^Z_{\rm mod} = \sqrt{\frac{Z_Z}{Z_z}} \sqrt{\frac{M_Z^2 - \delta M_Z^2}{M_Z^2}}, \qquad C^W_{\rm mod} = \sqrt{\frac{Z_W}{Z_w}} \sqrt{\frac{M_W^2 - \delta M_W^2}{M_W^2}}. \tag{13}$$

Here the wave function renormalization constants of the gauge bosons (Z and $W^\pm$) and the Nambu-Goldstone bosons (z and $w^\pm$)are denoted by Z_Z, Z_W, Z_z and Z_w, respectively. The origin of Eq. (13) may be explained in the following very

intuitive way. The factor $\sqrt{Z_Z/Z_z}$ and $\sqrt{Z_W/Z_w}$ compensates the difference in the external-line renormalization between Nambu-Goldstone and the gauge bosons. The presence of δM_Z^2 and δM_W^2 in Eq. (13) is due to the fact that gauge boson masses appearing in the longitudinal polarization vectors in the full theory are simply of kinematical origin irrelevant to renormalization, while they appear, in the Higgs-Goldstone systems, as coupling constants that are subject to renormalization. Thus the presence of δM_Z^2 and δM_W^2 in (13) may be undestood as making up this difference.

Toussaint [20] once calculated δM_W^2 and δM_Z^2 at the one-loop level in the two Higgs doublet model, assuming that all the Higgs masses are much greater than the gauge bosons'. For completeness, we would like to include the effects due to the top quark mass since it is of interest to see whether the largeness of the top quark mass might be just on the verge of affecting the Higgs decays. After including the top quark mass effects, Toussaint's results on the vector boson mass renormalization are modified into

$$\begin{aligned}\frac{\delta M_Z^2}{M_Z^2} &= \frac{1}{(4\pi)^2 v^2}\left\{\frac{1}{2}(m_H^2+m_h^2+m_A^2)\right.\\ &+\cos^2(\alpha-\beta)\frac{m_A^2 m_H^2}{m_H^2-m_A^2}\ln\frac{m_A^2}{m_H^2}+\sin^2(\alpha-\beta)\frac{m_A^2 m_h^2}{m_h^2-m_A^2}\ln\frac{m_A^2}{m_h^2}\\ &\left.+2N_C m_t^2(\frac{-2}{D-4}-\gamma_E-\ln\frac{m_t^2}{4\pi\mu^2})\right\}, \qquad (14)\end{aligned}$$

$$\begin{aligned}\frac{\delta M_W^2}{M_W^2} &= \frac{1}{(4\pi)^2 v^2}\left\{\frac{1}{2}(2m_G^2+m_H^2+m_h^2+m_A^2)+\frac{m_G^2 m_A^2}{m_A^2-m_G^2}\ln\frac{m_G^2}{m_A^2}\right.\\ &+\cos^2(\alpha-\beta)\frac{m_G^2 m_H^2}{m_H^2-m_G^2}\ln\frac{m_G^2}{m_H^2}+\sin^2(\alpha-\beta)\frac{m_G^2 m_h^2}{m_h^2-m_G^2}\ln\frac{m_G^2}{m_h^2}\\ &\left.+2N_C m_t^2(\frac{-2}{D-4}+\frac{1}{2}-\gamma_E-\ln\frac{m_t^2}{4\pi\mu^2})\right\}. \qquad (15)\end{aligned}$$

Here N_C is the number of colors ($N_C=3$), μ is the mass scale appearing in the D-dimensional regularization method, and here and hereafter we neglect the bottom quark mass effects.

To compute the wave function renormalization constants Z_z and Z_w of the Nambu-Goldstone bosons and in particular the quark contributions to them, we have summarized in Appendix A and Table 1 the Yukawa couplings of the top and bottom quarks to various scalar particles. There are two models of the Yukawa coupling, so-called model I [21] and model II [22]. In model I, both top and bottom quarks receive their masses from only one of the two Higgs doublets, say, Φ_2. In the model II, on the other hand, the top quark mass comes from Φ_2, and the bottom quark mass from Φ_1. As we see from the list of the couplings, there exsits little difference between these two models as far as the bottom quark mass is negligibly small. Setting $m_b \cong 0$, we will proceed hereafter without dicriminating the two models.

The wave function renormalization constants Z_z, and Z_w of the Nambu-Goldstone bosons are also easily extracted from the calculations in Ref. [20] together with those in Appendix C, namely,

$$
\begin{aligned}
Z_z &= 1-\frac{1}{(4\pi)^2v^2}\Bigg\{\frac{1}{2}(m_H^2+m_h^2+m_A^2) \\
&\quad +\cos^2(\alpha-\beta)\frac{m_A^2m_H^2}{m_H^2-m_A^2}\ln\frac{m_A^2}{m_H^2}+\sin^2(\alpha-\beta)\frac{m_A^2m_h^2}{m_h^2-m_A^2}\ln\frac{m_A^2}{m_h^2} \\
&\quad -2N_Cm_t^2\Big(\frac{2}{D-4}+\gamma_E+\ln\frac{m_t^2}{4\pi\mu^2}\Big)\Bigg\}, \qquad (16)
\end{aligned}
$$

$$
\begin{aligned}
Z_w &= 1-\frac{1}{(4\pi)^2v^2}\Bigg\{\frac{1}{2}(2m_G^2+m_H^2+m_h^2+m_A^2)+\frac{m_G^2m_A^2}{m_A^2-m_G^2}\ln\frac{m_G^2}{m_A^2} \\
&\quad +\cos^2(\alpha-\beta)\frac{m_G^2m_H^2}{m_H^2-m_G^2}\ln\frac{m_G^2}{m_H^2}+\sin^2(\alpha-\beta)\frac{m_G^2m_h^2}{m_h^2-m_G^2}\ln\frac{m_G^2}{m_h^2} \\
&\quad -2N_Cm_t^2\Big(\frac{2}{D-4}+\gamma_E+\ln\frac{m_t^2}{4\pi\mu^2}-1\Big)\Bigg\}. \qquad (17)
\end{aligned}
$$

The wave function renormalization constants of the W and Z bosons, on the other hand, are given by $Z_Z = 1+\mathcal{O}(M_Z^2/v^2)$, and $Z_W = 1+\mathcal{O}(M_W^2/v^2)$, and may be set equal to unity in our approximation scheme. In summary, the modification factors

in (13) turn out to be

$$C^Z_{\rm mod} \cong 1, \qquad C^W_{\rm mod} \cong 1 + \frac{1}{(4\pi)^2 v^2} \cdot \frac{N_C m_t^2}{2}. \tag{18}$$

4. The Higgs-Goldstone system

Being equipped with the machinery of the equivalence theorem, we are now interested in the radiative corrections to the processes $H \to zz$ and $H \to w^+w^-$. Since we assume that all the Higgs boson masses are much greater than M_Z and M_W, internal loops are also dominated by Higgs and Nambu-Goldstone bosons (plus top quark).

Before starting to calculate the radiative corrections, we have to go somewhat in detail on the structure of the counterterms, since there are subtleties with regard to the field mixing. The interaction Lagrangian relevant to Hzz and Hw^+w^- vertices is extracted from the Higgs potential (1)

$$\mathcal{L}_{Hzz} = \frac{m_H^2}{2v} \sin(\alpha - \beta) Hzz, \tag{19}$$

$$\mathcal{L}_{Hww} = \frac{m_H^2}{v} \sin(\alpha - \beta) Hw^+w^-. \tag{20}$$

The counterterms, $\delta\mathcal{L}_{Hzz}$, and $\delta\mathcal{L}_{Hww}$, consist of two parts, *i.e.*,

$$\delta\mathcal{L}_{Hzz} = \delta\mathcal{L}^{(1)}_{Hzz} + \delta\mathcal{L}^{(2)}_{Hzz}, \qquad \delta\mathcal{L}_{Hww} = \delta\mathcal{L}^{(1)}_{Hww} + \delta\mathcal{L}^{(2)}_{Hww}. \tag{21}$$

The first one is obtained simply by replacing the parameters in (19) and (20) as $m_H^2 \to m_H^2 - \delta m_H^2$, $\alpha \to \alpha - \delta\alpha$, $\beta \to \beta - \delta\beta$, $v \to v - \delta v$. We thus obtain

$$\begin{aligned} \delta\mathcal{L}^{(1)}_{Hzz} &= \left\{ -\frac{\delta m_H^2}{m_H^2} + \frac{\delta v}{v} \right\} \frac{m_H^2}{2v} \sin(\alpha - \beta) Hzz \\ &\quad - \frac{m_H^2}{2v} (\delta\alpha - \delta\beta) \cos(\alpha - \beta) Hzz, \end{aligned} \tag{22}$$

$$
\begin{aligned}
\delta\mathcal{L}^{(1)}_{Hww} &= \left\{-\frac{\delta m_H^2}{m_H^2}+\frac{\delta v}{v}\right\}\frac{m_H^2}{v}\sin(\alpha-\beta)Hw^+w^- \\
&\quad -\frac{m_H^2}{v}(\delta\alpha-\delta\beta)\cos(\alpha-\beta)Hw^+w^-. \qquad (23)
\end{aligned}
$$

The second one is rather complicated due to field mixing effects between $H \leftrightarrow h$, $z \leftrightarrow A$, and $w^\pm \leftrightarrow G^\pm$ pairs. These mixing effects are described by 2×2 wave function renormalization matrices together with the mixing angle renormalization, $\delta\alpha$ and $\delta\beta$. The renormalization is fullfilled by the replacement

$$
\begin{pmatrix} h \\ H \end{pmatrix} \to \begin{pmatrix} \sqrt{Z_h} & \sqrt{Z_{hH}} \\ \sqrt{Z_{Hh}} & \sqrt{Z_H} \end{pmatrix} \begin{pmatrix} \cos\delta\alpha & -\sin\delta\alpha \\ \sin\delta\alpha & \cos\delta\alpha \end{pmatrix} \begin{pmatrix} h \\ H \end{pmatrix}, \qquad (24)
$$

$$
\begin{pmatrix} z \\ A \end{pmatrix} \to \begin{pmatrix} \sqrt{Z_z} & \sqrt{Z_{zA}} \\ \sqrt{Z_{Az}} & \sqrt{Z_A} \end{pmatrix} \begin{pmatrix} \cos\delta\beta & -\sin\delta\beta \\ \sin\delta\beta & \cos\delta\beta \end{pmatrix} \begin{pmatrix} z \\ A \end{pmatrix}, \qquad (25)
$$

$$
\begin{pmatrix} w \\ G \end{pmatrix} \to \begin{pmatrix} \sqrt{Z_w} & \sqrt{Z_{wG}} \\ \sqrt{Z_{Gw}} & \sqrt{Z_G} \end{pmatrix} \begin{pmatrix} \cos\delta\beta & -\sin\delta\beta \\ \sin\delta\beta & \cos\delta\beta \end{pmatrix} \begin{pmatrix} w \\ G \end{pmatrix}. \qquad (26)
$$

Because of these mixings, the counterterms for the Hzz vertex are also produced from the hzz and HAz terms in the Lagrangian, *i.e.*,

$$
\mathcal{L}_{hzz} = -\frac{m_h^2}{2v}\cos(\alpha-\beta)hzz, \qquad (27)
$$

$$
\mathcal{L}_{HAz} = -\frac{m_H^2-m_A^2}{v}\cos(\alpha-\beta)HAz. \qquad (28)
$$

Similarly, the interactions coming from the potential (1)

$$
\mathcal{L}_{hww} = -\frac{m_h^2}{v}\cos(\alpha-\beta)hw^+w^-, \qquad (29)
$$

$$
\mathcal{L}_{HGw} = -\frac{m_H^2-m_G^2}{v}\cos(\alpha-\beta)(Hw^+G^- + Hw^-G^+), \qquad (30)
$$

provide some of the counterterms for Hw^+w^- interaction. We will restrict our consideration to the lowest-order loop-corrections and hence keep only linear terms

in $\delta\alpha$ and $\delta\beta$. It is in fact straightforward by the replacement (24), (25) and (26) in the interactions (27)-(30), to reach the following set of counterterms which are to be added to (22) and (23);

$$\begin{aligned}\delta\mathcal{L}^{(2)}_{Hzz} &= \left\{(\sqrt{Z_H}-1)+(Z_z-1)\right\}\frac{m_H^2}{2v}\sin(\alpha-\beta)Hzz \\ &\quad -(\sqrt{Z_{hH}}-\delta\alpha)\frac{m_h^2}{2v}\cos(\alpha-\beta)Hzz \\ &\quad -(\sqrt{Z_{Az}}+\delta\beta)\frac{m_H^2-m_A^2}{v}\cos(\alpha-\beta)Hzz,\end{aligned} \tag{31}$$

$$\begin{aligned}\delta\mathcal{L}^{(2)}_{Hww} &= \left\{(\sqrt{Z_H}-1)+(Z_w-1)\right\}\frac{m_H^2}{v}\sin(\alpha-\beta)Hw^+w^- \\ &\quad -(\sqrt{Z_{hH}}-\delta\alpha)\frac{m_h^2}{v}\cos(\alpha-\beta)Hw^+w^- \\ &\quad -2(\sqrt{Z_{Gw}}+\delta\beta)\frac{m_H^2-m_G^2}{v}\cos(\alpha-\beta)Hw^+w^-.\end{aligned} \tag{32}$$

It is by now clear what kinds of two-point functions are to be computed in order to complete our counterterms. Hereafter we will use the notations $\Pi_{ij}(p^2)$ for the two point functions corresponding to Fig. 1, where indices i and j denote either one of the scalar field, H, h, z, w, A or G. Internal particles in Fig. 1(a) and 1(b) are given in Table 2. Calculations of some of the two-point functions are sketched in Appendices C and D. The basic quantities in the counterterms are summarized as follows:

$$\begin{aligned}\delta m_H^2 &= \mathrm{Re}\left(\Pi_{HH}(m_H^2)\right), \qquad Z_H = 1+\mathrm{Re}\left(\Pi'_{HH}(m_H^2)\right), \\ Z_z &= 1+\mathrm{Re}\left(\Pi'_{zz}(0)\right), \qquad Z_w = 1+\mathrm{Re}\left(\Pi'_{ww}(0)\right).\end{aligned} \tag{33}$$

Here the renormalization is done on the mass shell, namely, at $p^2=m_H^2$ for Π_{HH} and $p^2=0$ for Π_{zz} and Π_{ww}. The divergences in the mixing of the two-point function Π_{hH} are taken care of by the counterterms $\delta\alpha$, $\sqrt{Z_{hH}}$ and $\sqrt{Z_{Hh}}$. In general, Z_{hH}

and Z_{Hh} are not bound to be identical. We will, however, impose $Z_{hH} = Z_{Hh}$ just for simplicity. If the subtraction is carried out at $p^2 = m_H^2$, they are determined by

$$\delta\alpha = \frac{1}{m_h^2 - m_H^2}\mathrm{Re}\left(\Pi_{hH}(m_H^2)\right) + \frac{1}{2}\mathrm{Re}\left(\Pi'_{hH}(m_H^2)\right), \tag{34}$$

$$\sqrt{Z_{hH}} = \sqrt{Z_{Hh}} = \frac{1}{2}\mathrm{Re}\left(\Pi'_{hH}(m_H^2)\right). \tag{35}$$

In the same way, the remainimg counterterms are determined by

$$\delta\beta = -\frac{1}{m_A^2}\mathrm{Re}\left(\Pi_{zA}(0)\right) - \frac{1}{2}\mathrm{Re}\left(\Pi'_{zA}(0)\right) = -\frac{1}{2}\mathrm{Re}\left(\Pi'_{zA}(0)\right), \tag{36}$$

$$\sqrt{Z_{zA}} = \sqrt{Z_{Az}} = \frac{1}{2}\mathrm{Re}\left(\Pi'_{zA}(0)\right), \qquad \sqrt{Z_{wG}} = \sqrt{Z_{wG}} = \frac{1}{2}\mathrm{Re}\left(\Pi'_{wG}(0)\right), \tag{37}$$

In Eq. (36), we have used the fact that $\Pi_{zA}(0)$ vanishes as a consequence of the Nambu-Goldstone theorem. The renormalization of β could be done alternatively by using $\Pi_{wG}(0)$. The difference between $\Pi_{zA}(0)$ and $\Pi_{wG}(0)$ is of course finite and infinities may be eliminated no matter whichever choice we would take. The use of Π_{zA} instead of Π_{wG} to determine $\delta\beta$ is just our convention.

We should add a few words as to the counterterm δv to the vacuuum expectation value. In the on-shell renormalization scheme [23] in which M_Z, M_W, and the fine structure constant $(e^2/4\pi)$ are physical input parameters, δv is determined by

$$\frac{\delta v}{v} = \frac{1}{2}\left(\frac{\delta M_W^2}{M_W^2} - \frac{\delta M_Z^2}{M_Z^2} + \frac{\delta M_Z^2 - \delta M_W^2}{M_Z^2 - M_W^2} - \frac{\delta e^2}{e^2}\right). \tag{38}$$

There is an alternative way which is referred to as modified on-shell scheme [24] and uses the muon decay constant instead of M_W as an input. This scheme was used in our previous work [5]. We have confirmed that the difference between the two prescriptions is small numerically. Just for definiteness, we use (38) in our numerical calculations.

5. Loop corrections to the vertices

We are now in a position to carry out the loop calculations of the Hzz vertex together with Hw^+w^-'s. The Feynman diagrams contributing to the renormalization of the Hzz vertex are depicted in Fig. 2. They are classified into scalar part and top quark part:

$$\Gamma_{Hzz}(p^2) = \Gamma^{(\mathrm{scalar})}_{Hzz}(p^2) + N_C \Gamma^{(\mathrm{quark})}_{Hzz}(p^2). \tag{39}$$

The scalar loop contributions are divided further into three terms

$$\Gamma^{(\mathrm{scalar})}_{Hzz}(p^2) = \Gamma^{(1)}_{Hzz} + \Gamma^{(2)}_{Hzz} + \Gamma^{(3)}_{Hzz} \tag{40}$$

according to the types of Feynman diagrams in Fig. 2. Internal particles in Fig. 2 are listed in Table 3.

The calculation is tedious but straightforward and we just record the results. The Feynman diagrams corresponding to Fig. 2(a) are summed up to:

$$\begin{aligned}
\Gamma^{(1)}_{Hzz} \ = \ & -\frac{m_H^6}{v^3}\sin^3(\alpha-\beta) g(p^2,0,0,m_H^2) \\
& -\frac{m_h^4 m_H^2}{v^3}\sin(\alpha-\beta)\cos^2(\alpha-\beta) g(p^2,0,0,m_h^2) \\
& +\frac{1}{v^3}\left\{ m_H^2\left(\frac{\cos\alpha\cos^2\beta}{\sin\beta} - \frac{\sin\alpha\sin^2\beta}{\cos\beta}\right) - 2m_A^2\sin(\alpha-\beta)\right\} \\
& \times\left\{ (m_h^2-m_A^2)^2\sin^2(\alpha-\beta) g(p^2,m_A^2,m_A^2,m_h^2) \right. \\
& \left. +(m_H^2-m_A^2)^2\cos^2(\alpha-\beta) g(p^2,m_A^2,m_A^2,m_H^2)\right\} \\
& +\frac{2m_h^2(m_H^2-m_A^2)(m_h^2-m_A^2)}{v^3}\sin(\alpha-\beta)\cos^2(\alpha-\beta) g(p^2,0,m_A^2,m_h^2) \\
& -\frac{2m_H^2(m_H^2-m_A^2)^2}{v^3}\sin(\alpha-\beta)\cos^2(\alpha-\beta) g(p^2,0,m_A^2,m_H^2) \\
& +\frac{3m_H^6}{v^3}\sin^2(\alpha-\beta)\left(\frac{\cos^3\alpha}{\sin\beta} - \frac{\sin^3\alpha}{\cos\beta}\right) g(p^2,m_H^2,m_H^2,0) \\
& +\frac{3m_H^2(m_H^2-m_A^2)^2}{v^3}\cos^2(\alpha-\beta)\left(\frac{\cos^3\alpha}{\sin\beta} - \frac{\sin^3\alpha}{\cos\beta}\right) g(p^2,m_H^2,m_H^2,m_A^2)
\end{aligned}$$

$$
\begin{aligned}
&-\frac{2m_h^2 m_H^2(2m_H^2+m_h^2)}{v^3}\sin(\alpha-\beta)\cos^2(\alpha-\beta)\frac{\sin 2\alpha}{\sin 2\beta}g(p^2,m_H^2,m_h^2,0)\\
&+\frac{2(m_h^2-m_A^2)(m_H^2-m_A^2)(2m_H^2+m_h^2)}{v^3}\sin(\alpha-\beta)\cos^2(\alpha-\beta)\frac{\sin 2\alpha}{\sin 2\beta}\\
&\times g(p^2,m_H^2,m_h^2,m_A^2)\\
&+\frac{m_h^4(m_H^2+2m_h^2)}{v^3}\sin(\alpha-\beta)\cos^2(\alpha-\beta)\frac{\sin 2\alpha}{\sin 2\beta}g(p^2,m_h^2,m_h^2,0)\\
&+\frac{(m_h^2-m_A^2)^2(m_H^2+2m_h^2)}{v^3}\sin^3(\alpha-\beta)\frac{\sin 2\alpha}{\sin 2\beta}g(p^2,m_h^2,m_h^2,m_A^2).
\end{aligned}
\tag{41}
$$

Each term of the above expression is easily identified with the corresponding Feynman diagrams just by looking at the function $g(p^2,m_1^2,m_2^2,m_3^2)$ defined in Appendix B. Those diagrams of the type of Fig. 2(b) are, on the other hand, expressed by the integral $f_2(p^2,m_1^2,m_2^2)$ which is also defined in Appendix B. They turn out to be:

$$
\begin{aligned}
\Gamma_{Hzz}^{(2)} &= \frac{5m_H^2}{2v^3}\left\{m_h^2\cos^2(\alpha-\beta)+m_H^2\sin^2(\alpha-\beta)\right\}\sin(\alpha-\beta)f_2(p^2,0,0)\\
&-\frac{1}{2v^3}\left\{m_H^2(\frac{\cos\alpha\cos^2\beta}{\sin\beta}-\frac{\sin\alpha\sin^2\beta}{\cos\beta})-2m_A^2\sin(\alpha-\beta)\right\}\\
&\times\left\{3m_h^2\sin^2(\alpha-\beta)+3m_H^2\cos^2(\alpha-\beta)\right.\\
&\left.+\frac{\sin 2\alpha}{\sin 2\beta}(m_h^2-m_H^2)\right\}f_2(p^2,m_A^2,m_A^2)\\
&-\frac{2}{v^3}\left\{m_H^2(\frac{\cos\alpha\cos^2\beta}{\sin\beta}-\frac{\sin\alpha\sin^2\beta}{\cos\beta})-2m_G^2\sin(\alpha-\beta)\right\}\\
&\times\left\{\frac{1}{2}m_h^2\sin^2(\alpha-\beta)+\frac{1}{2}m_H^2\cos^2(\alpha-\beta)\right.\\
&\left.+\frac{1}{2}\frac{\sin 2\alpha}{\sin 2\beta}(m_h^2-m_H^2)+m_G^2\right\}f_2(p^2,m_G^2,m_G^2)\\
&-\frac{2}{v^3}(m_h^2-m_H^2)(m_H^2-m_G^2)\sin(\alpha-\beta)\cos^2(\alpha-\beta)f_2(p^2,m_G^2,0)\\
&-\frac{3}{v^3}(m_h^2-m_H^2)(m_H^2-m_A^2)\sin(\alpha-\beta)\cos^2(\alpha-\beta)f_2(p^2,m_A^2,0)\\
&-\frac{3m_H^2}{2v^3}\left\{\left((m_h^2-m_H^2)\frac{\sin 2\alpha}{\sin 2\beta}+2m_A^2\right)\cos^2(\alpha-\beta)+m_H^2\right\}
\end{aligned}
$$

$$
\begin{aligned}
&\times\left(\frac{\cos^3\alpha}{\sin\beta}-\frac{\sin^3\alpha}{\cos\beta}\right)f_2(p^2,m_H^2,m_H^2)\\
&-\frac{1}{v^3}\left\{(m_h^2-m_H^2)\frac{\sin 2\alpha}{\sin 2\beta}+2m_A^2\right\}(m_h^2+2m_H^2)\\
&\times\sin(\alpha-\beta)\cos^2(\alpha-\beta)\frac{\sin 2\alpha}{\sin 2\beta}f_2(p^2,m_H^2,m_h^2)\\
&-\frac{1}{2v^3}\left\{\left((m_h^2-m_H^2)\frac{\sin 2\alpha}{\sin 2\beta}+2m_A^2\right)\sin^2(\alpha-\beta)+m_h^2\right\}\\
&\times(m_H^2+2m_h^2)\sin(\alpha-\beta)\frac{\sin 2\alpha}{\sin 2\beta}f_2(p^2,m_h^2,m_h^2).
\end{aligned}
\tag{42}
$$

Finally we come to the sum of Feynman diagrams of the type Fig. 2(c):

$$
\begin{aligned}
\Gamma^{(3)}_{Hzz} \;=\;& -\frac{2m_h^2}{v^3}\left\{(m_h^2-m_H^2)\frac{\sin 2\alpha}{\sin 2\beta}+2m_A^2\right\}\\
&\times\sin(\alpha-\beta)\cos^2(\alpha-\beta)f_2(0,m_h^2,0)\\
&+\frac{2m_H^2}{v^3}\Bigg\{(m_h^2\frac{\sin 2\alpha}{\sin 2\beta}+2m_A^2)\sin(\alpha-\beta)\cos^2(\alpha-\beta)\\
&+m_H^2(\frac{\sin^3\alpha}{\cos\beta}-\frac{\cos^3\alpha}{\sin\beta})\sin^2(\alpha-\beta)\Bigg\}f_2(0,m_H^2,0)\\
&-\frac{2(m_h^2-m_A^2)}{v^3}\Bigg\{\frac{\sin 2\alpha}{\sin 2\beta}(m_h^2\sin^2(\alpha-\beta)+m_H^2\cos^2(\alpha-\beta))\\
&-m_A^2\cos(2\alpha-2\beta)\Bigg\}\sin(\alpha-\beta)f_2(0,m_A^2,m_h^2)\\
&-\frac{m_H^2-m_A^2}{v^3}\Bigg\{(m_h^2\frac{\sin 2\alpha}{\sin 2\beta}+2m_A^2)\sin(2\alpha-2\beta)\\
&+2m_H^2(\frac{\cos^3\alpha}{\sin\beta}-\frac{\sin^3\alpha}{\cos\beta})\cos(\alpha-\beta)\Bigg\}\cos(\alpha-\beta)f_2(0,m_A^2,m_H^2).
\end{aligned}
\tag{43}
$$

The top quark contributions are obtained by using the Yukawa couplings listed in Table 1:

$$
\Gamma^{(\mathrm{quark})}_{Hzz}=\frac{4m_t^4}{v^3}\frac{\cos\alpha}{\sin\beta}\left\{2f_2(p^2,m_t^2,m_t^2)-p^2g(p^2,m_t^2,m_t^2,m_t^2)\right\}. \tag{44}
$$

The loop corrections to the Hw^+w^- vertex go in the same way and we again separate the contributions into scalar and top quark parts,

$$\Gamma_{Hww}(p^2) = \Gamma^{(\mathrm{scalar})}_{Hww}(p^2) + N_C \Gamma^{(\mathrm{quark})}_{Hww}(p^2). \tag{45}$$

The scalar contributions $\Gamma^{(\mathrm{scalar})}_{Hww}(p^2)$ were computed in our previous paper (see Eq. (31) in Ref. [5]), and we need not reproduce them here. The top and bottom quark part is given, if we set $m_b = 0$, by

$$\begin{aligned}\Gamma^{(\mathrm{quark})}_{Hww} &= \frac{4m_t^4}{v^3}\frac{\cos\alpha}{\sin\beta}\left\{f_2(p^2, m_t^2, m_t^2) + f_2(0, 0, m_t^2)\right\} \\ &\quad - \frac{4m_t^6}{v^3}\frac{\cos\alpha}{\sin\beta} g(p^2, m_t^2, m_t^2, 0).\end{aligned} \tag{46}$$

By adding the counterterm contributions we end up with the decay width formulae,

$$\Gamma(H \to Z_L Z_L) = \frac{1}{32\pi}\frac{1}{m_H}\sqrt{1 - \frac{4M_Z^2}{m_H^2}}|\mathcal{M}_{Hzz}(p^2 = m_H^2)|^2 |C^Z_{\mathrm{mod}}|^4, \tag{47}$$

$$\Gamma(H \to W_L^+ W_L^-) = \frac{1}{16\pi}\frac{1}{m_H}\sqrt{1 - \frac{4M_W^2}{m_H^2}}|\mathcal{M}_{Hww}(p^2 = m_H^2)|^2 |C^W_{\mathrm{mod}}|^4, \tag{48}$$

where the invariant amplitudes are given by

$$\begin{aligned}\mathcal{M}_{Hzz}(p^2) &= \Gamma_{Hzz}(p^2) + \frac{1}{v}\cos(\alpha-\beta)\mathrm{Re}\left(\Pi_{hH}(m_H^2)\right) \\ &\quad -\frac{1}{v}\sin(\alpha-\beta)\mathrm{Re}\left(\Pi_{HH}(m_H^2)\right) \\ &\quad +\left\{\frac{\delta v}{v} + \frac{1}{2}\mathrm{Re}\left(\Pi'_{HH}(m_H^2)\right) + Z_z\right\}\frac{m_H^2}{v}\sin(\alpha-\beta) \\ &\quad -\mathrm{Re}\left(\Pi'_{zA}(0) + \Pi'_{hH}(m_H^2)\right)\frac{m_H^2}{2v}\cos(\alpha-\beta),\end{aligned} \tag{49}$$

$$\begin{aligned}\mathcal{M}_{Hww}(p^2) &= \Gamma_{Hww}(p^2) + \frac{1}{v}\cos(\alpha-\beta)\mathrm{Re}\left(\Pi_{hH}(m_H^2)\right) \\ &\quad -\frac{1}{v}\sin(\alpha-\beta)\mathrm{Re}\left(\Pi_{HH}(m_H^2)\right)\end{aligned}$$

$$
\begin{aligned}
&+\left\{\frac{\delta v}{v}+\frac{1}{2}\mathrm{Re}\left(\Pi'_{HH}(m_H^2)\right)+Z_w\right\}\frac{m_H^2}{v}\sin(\alpha-\beta)\\
&-\mathrm{Re}\left(\Pi'_{hH}(m_H^2)+\Pi'_{wG}(0)\right)\frac{m_H^2}{2v}\cos(\alpha-\beta)\\
&+\mathrm{Re}\left(\Pi'_{wG}(0)-\Pi'_{zA}(0)\right)\frac{2m_G^2-m_H^2}{2v}\cos(\alpha-\beta).
\end{aligned}
\tag{50}
$$

These amplitudes are necessarily finite and the finiteness is a non-trivial check of the calculations.

6. Numerical analyses of the decay widths

Let us now analyze the decay width formulae numerically and look at their heavy Higgs boson mass-dependences. In doing so we have to select reasonable numbers for the set of parameters, m_H, m_G, m_A, m_h, α and β. There are four kinds of experimental information that we have to bear in our mind: (1) the measurement of the ρ-parameter [20, 25], (2) the neutral meson mixings ($K^0-\bar{K}^0$, $B^0-\bar{B}^0$, and $D^0-\bar{D}^0$) [26], (3) the recent measurement of the decays such as $B\to K^*(892)\gamma$ [27], (4) the ratios, $R_b=\Gamma(Z\to b\bar{b})/\Gamma(Z\to \text{hadrons})$, and the R_c counterpart [28].

It has been known rather well that the constraints on the deviation of the ρ-parameter from unity prohibits m_G to be much larger than or much smaller than either of neutral Higgs boson masses. These constraints on the Higgs masses, however, depend on the mixing angle, α and β. The simultaneous analysis on the masses and mixing angles would become much involved. For the purpose of getting an insight into a global picture of the m_G- and m_A-dependences of the decay widths, we may well vary the masses , m_G and m_A, a little beyond the ρ-parameter constraint. The data on neutral meson mixings and $b\to s\gamma$ decay rate both rule out small value of m_G and small $\tan\beta$. Grant [29] has confronted the two-doublet model with these experimental data simultaneously and made an overall analyses. The constraints he derived are $m_G>150-200$GeV and $\tan\beta>0.7$. We will give these values due

considerations in the following numerical analyses. The so-called $R_b - R_c$ crisis [28] , which might jeopardise the standard model, is our recent central concern. It seems, however, yet premature to draw definite conclusions from those data or implications to the two-doublet model. We will henceforth wait a little while for what comes next.

Besides these sets of experimental information, we have theoretical contraints on the masses in the two-doublet model. One of them is the triviality bound [30] and another is the tree unitarity bound [31]. The latter constraints lead to $m_H < 500$ GeV, $m_G < 870$ GeV, $m_h < 710$ GeV and $m_A < 1200$ GeV. The triviality bounds also provide roughly similar results. We will take these into our consideration as a guide of our parameter choices. We will set $m_H = 300$ GeV, and $m_h = 400$ GeV throughout our numerical calculations.

First of all, we look at the m_G- and m_A-dependences of $\Gamma(H \to Z_L Z_L)$ and $\Gamma(H \to W_L^+ W_L^-)$ for the following two cases,

Case I: $\tan\beta = 2$ and $\sin^2(\alpha - \beta) = 1$,

Case II: $\tan\beta = 10$ and $\sin^2(\alpha - \beta) = 0.5$.

The decay widths $\Gamma(H \to Z_L Z_L)$ and $\Gamma(H \to W_L^+ W_L^-)$ as a function of m_G depicted in Figs. 3 and 4 correspond to the above two cases. We vary m_G from 300 GeV to 1000 GeV, while m_A is set equal to 400, 600 and 800 GeV.

To get a rough idea of numerical values let us recall that these decay rates in the minimal standard model (MSM) with a single Higgs doublet give at the tree level

$$\begin{aligned} \Gamma^{\mathrm{MSM}}(H \to Z_L Z_L)|_{\mathrm{tree}} &= \frac{1}{32\pi}\frac{m_H^3}{v^2}\sqrt{1 - \frac{4M_Z^2}{m_H^2}}, \\ \Gamma^{\mathrm{MSM}}(H \to W_L^+ W_L^-)|_{\mathrm{tree}} &= \frac{1}{16\pi}\frac{m_H^3}{v^2}\sqrt{1 - \frac{4M_W^2}{m_H^2}}. \end{aligned} \tag{51}$$

For $m_H = 300$ GeV these formulae give $\Gamma^{\mathrm{MSM}}(H \to Z_L Z_L) = 3.5$GeV and $\Gamma^{\mathrm{MSM}}(H \to W_L^+ W_L^-) = 7.5$GeV. The tree level decay widths in the two Higgs

doublet model are suppressed by a factor $\sin^2(\alpha-\beta)$ compared with (51). To take into account the one-loop corrections in MSM amounts to multiplying (51) by the factor

$$\left|1+\frac{1}{4\pi^2}\frac{m_H^2}{v^2}\left(\frac{19}{16}-\frac{3\sqrt{3}\pi}{8}+\frac{5\pi^2}{48}\right)\right|^2. \tag{52}$$

This factor is about 1.013 for $m_H = 300$GeV and the radiative corrections have been said to be very small in MSM.

The situation in the two-Higgs doublet model could differ very much from this. The radiative corrections would be enhanced if m_G^2/v^2, m_A^2/v^2 and m_h^2/v^2 are large and could stand out from the lowest-order values. Our interest is to what extent they could supersede the tree-predictions. Figs. 3 and Fig. 4 give us typical patterns of the m_G- and m_A-dependences. One can immediately notice that the decay rate of $H \to Z_L Z_L$ depends on m_G and m_A surprisingly little in contrast to our naive expectation. We have also confirmed that the m_h dependence is very weak. We can hardly see any indication of power-behaviors w.r.t. heavier Higgs boson masses.

The decay width of $H \to W_L^+ W_L^-$ on the other hand exhibits some power-behavior. The one-loop corrections are potentially large to overcome the tree-level predictions. One thing that we should pay attention here is that, while the radiative corrections are large in general, they tend to become small for $m_G = m_A$. The apparent difference lying between the two decay modes into W- and Z-pairs is puzzling. If the isospin symmetry $SU(2)_V$ is exact, these two decay rates differ simply by a factor of two (up to the phase space difference) irrespectively of the choice of the parameters. In our two-Higgs doublet model, the isospin symmetry is broken by the λ_5-term in which we have necessarily to seek for the source of the difference.

The overall properties of the decay widths may be seen easier if one would draw the widths in the birds' eye views. Figs 5 and 6 give those of the decay width of $\Gamma(H \to Z_L Z_L)$ as a function of m_G and m_A, and Figs. 7 and 8 those of $\Gamma(H \to W_L^+ W_L^-)$. Figs. 5 and 7 correspond to the case (I) and Figs. 6 and 8 to the case

(II).

Figs. 5 and 6 show that the decay width of $H \to Z_L Z_L$ does not go far away from the tree-level values (3.5 GeV for Fig. 5 and 1.75 GeV for Fig. 6) for a wide range of parameters, 400GeV $< m_G, m_A <$ 1000GeV. This is rather in contrast with our naive expectation. Considering that m_G^2/v^2 and m_A^2/v^2 are greater than unity, the radiative corrections could instabilize the tree predictions. The reason for the almost flat behavior in Figs. 5 and 6 will be elucidated qualitatively in Sec. 7.

In the case of $H \to W_L^+ W_L^-$, on the other hand, we can immediately see in Fig. 7 a conspicuous ridge elongated along the line $m_G = m_A$. The width comes close to the tree level prediction along this line, *i.e.*, the radiative corrections tend to be minimized in the custodial $SU(2)_V$ symmetric limit. The corrections become larger and larger as we go off from this $m_G = m_A$ line. This reflects the fact that the radiative corrections entail the terms proportional to m_G^4, $m_G^2 m_A^2$ and m_A^4 that exceed the tree-values. The shape of the surface in Fig. 8 corrsponding to the case II differs from that in Fig. 7. The one-loop predictions go up and down depending on the choice of mass parameters. The point to be emphasized is that the line $m_G = m_A$ is again given a special meaning. The radiative corrections along this line tend to be minimized, the tree-level width being equal to 3.8 GeV.

To sum up our numerical computation, two questions have emerged naturally. One is why the radiative corrections to the decay rate of $H \to W_L^+ W_L^-$ are minimized for $m_G = m_A$. The other question is what explains the difference between the two decay rates and how. We have included the top quark mass effect in Figs. 3-8. The top contributions, however, modify the predictions of the widths only within a few per cent and are not very significant.

7. New screening theorem for Higgs vertex

To shed light on the two questions raised at the end of Sec. 6, we disentangle the leading terms among $\mathcal{M}_{Hww}(p^2 = m_H^2)$ and $\mathcal{M}_{Hzz}(p^2 = m_H^2)$. By leading terms we mean those containing m_G^2, m_A^2, and m_h^2, but not m_H^2 at all. The results are as follows:

$$\begin{aligned}
\mathcal{M}_{Hww}(m_H^2) \longrightarrow \quad & \frac{1}{(4\pi)^2 v^3} \sin(\alpha - \beta) \left\{ (m_G^2 - m_A^2) m_A^2 - m_G^2 m_A^2 \ln \frac{m_G^2}{m_A^2} \right\} \\
& +(\text{terms depending on the prescription for } \delta\beta) \\
& +\mathcal{O}(\frac{m_H^2 m_G^2}{v^3}, \frac{m_H^2 m_A^2}{v^3}, \text{ or } \frac{m_H^2 m_h^2}{v^3}),
\end{aligned} \tag{53}$$

$$\mathcal{M}_{Hzz}(m_H^2) \longrightarrow \quad \mathcal{O}(\frac{m_H^2 m_G^2}{v^3}, \frac{m_H^2 m_A^2}{v^3}, \text{ or } \frac{m_H^2 m_h^2}{v^3}). \tag{54}$$

The second line in (53) is due to the terms that arise by our choice of Π_{zA} instead of Π_{wG} in defining the renormalization $\delta\beta$, *i.e.*, Eq. (36). In other words, these come from the last line of Eq. (50).

This leading behavior derived by hand in (53) and (54) is of course consistent with our numerical computations. The decay into a W-pair behaves as fourth power of m_G and/or m_A, but does not contain m_h in (53). It is therefore insensitive to m_h. Furthermore the leading terms in (53) also disappear if we put $m_G = m_A$. Recall that the second line in (53) consists of those terms containing $m_G^2 - m_A^2$. The behavior (54) indicates that all of the leading terms just disappear after summing up all the diagrams together with counterterms and the decay rate $\Gamma(H \to Z_L Z_L)$ is fairly insensitive to m_G, m_A, or m_h.

From the viewpoint of the custodial $SU(2)_V$ symmetry and its breaking, it is natural to divide the decay amplitudes into two parts

$$\mathcal{M}_{Hww} = \mathcal{M}^S + \mathcal{M}^B_{Hww}, \tag{55}$$

$$\mathcal{M}_{Hzz} = \mathcal{M}^S + \mathcal{M}^B_{Hzz}. \tag{56}$$

Here $\mathcal{M}^S$ is the custodial symmetric part and $\mathcal{M}^B_{Hww}$ and $\mathcal{M}^B_{Hzz}$ are those due to the custodial symmetry breaking. As we mentioned in Sec. 2, the custodial

symmetry breaking occurs through the λ_5-coupling. This breaking effect shows up in two ways in the perturbative calculations. One is in the broken-symmetry in the Higgs-Goldstone couplings, the other in the different masses in G and A propagators. Both of these two effects are collected in the second terms of (55) and (56). The symmetric part $\mathcal{M}^S$ are obtained simply by replacing m_A by m_G everywhere in $\mathcal{M}_{Hww}$ and $\mathcal{M}_{Hzz}$.

The leading behavior shown in (53) and (54) indicates that the custodial symmetric part $\mathcal{M}^S$ does not possess the fourth-power terms w.r.t. m_G, m_A or m_h. We may conclude that $\mathcal{M}^S$ is insensitive to the heavier Higgs boson masses. We are thus led to postulate a new screening theorem.

Theorem: There occurs a cancellation mechanism among the leading terms w.r.t. the heavier Higgs boson masses in the custodial $SU(2)_V$-symmetric limit in the radiative corrections to the decay rates of $H \to Z_L Z_L$ and $H \to W_L^+ W_L^-$.

It should be stressed that this theorem differs from Veltman's in that the Veltman's theorem has been proved for the oblique-type radiative corrections, while the above one is for the Higgs decay vertex. It should be also noticed that despite this difference the custodial symmetry is the key ingredient in both cases. Recall that the proof by Einhorn and Wudka relies heavily on the custodial symmetry. What we did in Sec. 6 and in (53) and (54) amounts to confirming the above theorem explicitly on the one-loop level by numerical calculations and by hand, respectively. We do not present here a general proof of our theorem and we should not go so far as to say anything definite as to the two-loop and higher-order cases, only mentionimg the following: the fact that the custodial symmetry is playing the key role in the Veltman's theorem is suggestive of the validity of our theorem beyond one-loop. It is also extremely tempting to speculate that there could occur a similar cancellation mechanism in other decay modes of the H-boson. We will come to this important issue in our future publications.

Now let us move on to the custodial symmetry breaking part. The absence of the leading terms in $\mathcal{M}^S$, means that the sensitive properties of the amplitude all come from $\mathcal{M}^B_{Hww}$ and $\mathcal{M}^B_{Hzz}$. In the W-pair dacay, $\mathcal{M}^B_{Hww}$ vanishes for $m_G = m_A$ that corresponds to the flat direction shown in Figs. 7 and 8. Thus the answer to the first question raised in Sec. 6, namely, the minimization of the radiative corrections along the line $m_G = m_A$, becomes almost self-evident once we accept our screening theorem.

The second question is what explains the apparent difference lying between decays into W- and Z-pairs. As we remarked before, the custodial symmetry breaking effects appear in the Higgs couplings on one hand, and the different G and A masses in the propagators on the other. Let us concentrate on the former, and in particular on the broken-symmetry in the Higgs coupling triggered by the λ_5-term in the potential

$$\begin{aligned}\lambda_5(\mathrm{Im}\Phi_1^\dagger\Phi_2)^2 \quad = \quad & -\frac{\lambda_5}{4}\Big[(w^+G^- - G^+w^-) \\ & +iA\left\{v + h\cos(\alpha-\beta) - H\sin(\alpha-\beta)\right\} \\ & -iz\{h\sin(\alpha-\beta) + H\cos(\alpha-\beta)\}\Big]^2. \end{aligned} \tag{57}$$

A close look at this expression shows that there exists an interaction $(w^+G^- - w^-G^+)A$, while another possible term

$$(w^+G^- - w^-G^+)z \tag{58}$$

is missing. The absence of this triple interaction indicates that there are some Feynman diagrams contributing to $H \to w^+w^-$ which do not have a conterpart in $H \to zz$. Actually the z-interactions with charged scalars are peculiar: not only (58) but also terms such as

$$w^+w^-z, \quad G^+G^-z, \quad (w^+G^- + w^-G^+)z, \tag{59}$$

are absent in the whole of the potential $V(\Phi_1, \Phi_2)$. These are all forbidden interaction vertices if either G-parity or CP-invariance is exact.

This peculiarity leads to a considerable simplification in tracing the m_G dependence in $\mathcal{M}_{Hzz}$ on which we now focus our attentions. Neither of $\Gamma^{(1)}_{Hzz}$ nor $\Gamma^{(3)}_{Hzz}$has the m_G-dependence at all, as one can make sure of in Eqs. (41) and (43). This is simply because there is no Feynman diagram of the type of Fig. 2(a) or Fig. 2(c) involving $G^{\pm}$. In the heavy Higgs mass limit ($m_G, m_A, m_h \gg m_H$), the m_G-dependence of the amplitude of the $Z_L Z_L$-decay is governed by the combination

$$\Gamma^{(2)}_{Hzz} + \frac{1}{v}\cos(\alpha-\beta)\mathrm{Re}(\Pi_{hH}(m_H^2)) - \frac{1}{v}\sin(\alpha-\beta)\mathrm{Re}(\Pi_{HH}(m_H^2)). \tag{60}$$

All the other terms in (49) are sub-leading in the heavy Higgs mass limit . The m_G-dependence comes about from the Feynman diagrams, Fig. 1(a) and Fig. 2(b) in which either of wG, or GG pair is encircling.

Now notice that the last two terms in (60) join together to produce the two-point function connecting the state $|H>$ with the linear combination

$$\frac{1}{v}\cos(\alpha-\beta)|h> - \frac{1}{v}\sin(\alpha-\beta)|H> . \tag{61}$$

An important point to be observed here is that the couplings of the combination (61) to wG- and GG-pairs are exactly of the same strength with opposite sign as the quartic $zz(w^+G^- + w^-G^+)$ and zzG^+G^- couplings, respectively, that contribute to $\Gamma^{(2)}_{Hzz}$. Thus the leading terms containing m_G (*i.e.*, m_G^4/v^3, $m_G^2 m_A^2/v^3$ and $m_G^2 m_h^2/v^3$) all disappear in (60) and thus in $\mathcal{M}_{Hzz}$ as well.

The equality of the strength of quartic and triple couplings as described above may be undestood easily if we go over to the so-called Georgi-basis [32], in which the combinations

$$\begin{aligned}
&v + h\cos(\alpha-\beta) - H\sin(\alpha-\beta) + iz, \\
&h\sin(\alpha-\beta) + H\cos(\alpha-\beta) + iA
\end{aligned} \tag{62}$$

are taken from the outset. It is also to be mentioned that the equality of the quartic and triple couplings is necessary for the ultraviolet divergences to disappear in the combination (60).

Once the absence of leading terms of the form m_G^4/v^3, $m_G^2 m_h^2/v^3$ and $m_G^2 m_A^2/v^3$ is established in $\mathcal{M}_{Hzz}^B$, then it is almost trivial that those proportional to m_A^4/v^3 and $m_A^2 m_h^2/v^3$ are also absent. This is because the leading terms in $\mathcal{M}_{Hzz}^B$ must vanish by setting $m_G = m_A$ and this is possible only when those terms are not there.

In this way we are able to disentangle so many Feynman diagrams and to trace the origin of the difference lying between $\mathcal{M}_{Hww}$ and $\mathcal{M}_{Hzz}$. We are now convinced how and why the moderate behaviors of $\Gamma(H \to Z_L Z_L)$ are seen in Figs. 5 and 6.

8. Summary and Discussions

In the present paper, we have studied one-loop radiative corrections to the Higgs boson (H) decay into longitudinal gauge boson pairs, $H \to Z_L Z_L$ and $H \to W_L^+ W_L^-$. The two Higgs doublet model has been considered throughout and H is assumed to be much heavier than the weak gauge bosons. A particular attention has been paid to the non-decoupling effects due to the other Higgs bosons which are assumed to be all heavier than H.

Our numerical analyses on the decay rate $H \to W_L^+ W_L^-$ show that the radiative corrections are potentially large. The larger the mass difference $m_G^2 - m_A^2$ becomes, the larger the deviation from the tree-predictions turns out to be. The point is that the radiative corrections tend to be minimized if $m_G^2 = m_A^2$, for which the custodial $SU(2)_V$-symmetry is recovered. This fact has led us to postulate a new screening theorem for Higgs vertices with reference to the custodial symmetry, which may be regarded as a generalization of the Veltman's theorem.

Although our generalized screening theorem has been confirmed only at the one-loop level for the particular decay processes, this type of theorem would hopefully play the role of a working-hypothesis in our future study. It will be of particular

interest to see if the custodial symmetry would have any relevance to the screening effect of heavy particles in other types of non-decoupling processes. Some examples to be examined include triple gauge boson couplings [33] and longitudinal gauge boson scatterings, $W_L^+W_L^- \to W_L^+W_L^-$ and $W_L^+W_L^- \to Z_L Z_L$. It is also important to see what happens in models other than the two-Higgs doublet model. We will come to these subjects in our future publications.

The decay rate $\Gamma(H \to Z_L Z_L)$ has turned out to be unexpectedly insensitive to heavier Higgs boson masses. The reason for this was elucidated in detail in Sec. 7. It is thus rather difficult to use this decay rate to get any signature of the masses of unknown heavy particles. Alternatively, however, by turning the tables around the decay $H \to Z_L Z_L$ could be useful to get information on the mixing angle α. Fig. 9 shows the α-dependence of this decay rate for $\tan\beta = 2$, $\tan\beta = 5$ and $\tan\beta = 10$. Measurements of the width with an accuracy on the order of a fraction of 1 GeV would enable us to determine the value of α.

Finally we would like to add a few rather peripheral remarks. As we mentioned in Sec. 2, we neglected the term $(\mu_{12}^2\Phi_1^\dagger\Phi_2 + \mu_{12}^{2*}\Phi_2^\dagger\Phi_1)$ which would break the discrete symmetry of the Higgs potential (1). By dropping this term, the heavy Higgs boson mass limit is rendered to be the same as the strong quartic couplings (see Eqs. (7)-(11)). Thus the non-decoupling effects are expected to be potentially large because of the strong couplings. If we would included $(\mu_{12}^2\Phi_1^\dagger\Phi_2 + \mu_{12}^{2*}\Phi_2^\dagger\Phi_1)$ as in minimally supersymmetric models, one may wonder whether the non-decoupling effects could be expected or not. In the presence of the mass scale μ_{12} in addition to v, the heavy Higgs mass limit does not always imply the strong quartic couplings. There exists a limit in which μ_{12}, m_h, m_A and m_G are all large while the quartic coulings are small. In such a case, the decoupling is expected from the begining and the two Higgs doublet model becomes similar to the minimal standard model at low energies [34].

The non-decoupling effects considered in this paper may be studied by the electro-weak chiral Lagrangian approach [35-37]. This method is powerful in the minimal standard model for systematic studies of low-energy manifestation of heavy particles. Whether this method is useful in the two Higgs doublet model as well is yet to be scrutinized and we leave it as an open problem.

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Appendix A

The Yukawa couplings of top and bottom quarks are given by

$$\mathcal{L}_{\rm Yukawa} = -\frac{\sqrt{2}m_b}{v\sin\beta}\bar{Q}_L\Phi_2 b_R - \frac{\sqrt{2}m_t}{v\sin\beta}\bar{Q}_L i\tau_2\Phi_2^* t_R + \text{h.c.}, \tag{63}$$

for model I and

$$\mathcal{L}_{\rm Yukawa} = -\frac{\sqrt{2}m_b}{v\cos\beta}\bar{Q}_L\Phi_1 b_R - \frac{\sqrt{2}m_t}{v\sin\beta}\bar{Q}_L i\tau_2\Phi_2^* t_R + \text{h.c.}, \tag{64}$$

for model II, where $Q_L = (t_L, b_L)^T$. More explicitly, these interactions are expanded by putting Eqs. (3)-(6) as follows;

$$\begin{aligned}
\mathcal{L}_{\rm Yukawa} \;=\; & -m_b\bar{b}b + C_1\bar{b}bH + C_2\bar{b}bh + C_3\bar{b}i\gamma_5 bz + C_4\bar{b}i\gamma_5 bA \\
& -m_t\bar{t}t + D_1\bar{t}tH + D_2\bar{t}th + D_3\bar{t}i\gamma_5 tz + D_4\bar{t}i\gamma_5 tA \\
& +E_1\left\{\bar{t}(1+\gamma_5)bw^+ + \bar{b}(1-\gamma_5)tw^-\right\} \\
& +E_2\left\{\bar{t}(1+\gamma_5)bG^+ + \bar{b}(1-\gamma_5)tG^-\right\} \\
& +F_1\left\{\bar{t}(1-\gamma_5)bw^+ + \bar{b}(1+\gamma_5)tw^-\right\} \\
& +F_2\left\{\bar{t}(1-\gamma_5)bG^+ + \bar{b}(1+\gamma_5)tG^-\right\}.
\end{aligned} \tag{65}$$

The coefficients in the above are tabulated in Table 1.

Appendix B

We define the following functions to express the various Feynman integrals:

$$\begin{aligned} f_1(m^2) &= \mu^{4-D}\int \frac{d^Dk}{(2\pi)^D}\frac{i}{k^2-m^2} \\ &= \frac{m^2}{(4\pi)^2}\left(\frac{2}{D-4}-1+\gamma_E+\ln\frac{m^2}{4\pi\mu^2}\right), \end{aligned} \tag{66}$$

$$f_2(p^2,m_1^2,m_2^2) = -i\mu^{4-D}\int \frac{d^Dk}{(2\pi)^D}\frac{i}{k^2-m_1^2}\frac{i}{(p-k)^2-m_2^2}, \tag{67}$$

$$g(p^2,m_1^2,m_2^2,m_3^2) = \int \frac{d^4k}{(2\pi)^4}\frac{i}{(k-p_1)^2-m_1^2}\frac{i}{(k+p_2)^2-m_2^2}\frac{i}{k^2-m_3^2}. \tag{68}$$

The function (68) can be expressed in terms of some combinations of the Spence function. Details are given in Appendix A of Ref. [5].

Appendix C

Here we would like to summarize the full expressions of the various two-point functions used in the text. Let us begin with $\Pi_{zA}(p^2)$ and $\Pi_{zz}(p^2)$, which are both vanishing at $p^2=0$ as a reflection of the Nambu-Goldstone theorem and are put in the following forms

$$\Pi_{zz}(p^2) = \hat{\Pi}_{zz}(p^2) - \hat{\Pi}_{zz}(0), \qquad \Pi_{zA}(p^2) = \hat{\Pi}_{zA}(p^2) - \hat{\Pi}_{zA}(0). \tag{69}$$

The functions $\hat{\Pi}_{zz}(p^2)$ and $\hat{\Pi}_{zA}(p^2)$ are given as a sum of scalar and quark contributions

$$\hat{\Pi}_{zz}(p^2) = \hat{\Pi}_{zz}^{(\mathrm{scalar})}(p^2)+\hat{\Pi}_{zz}^{(\mathrm{quark})}(p^2), \quad \hat{\Pi}_{zA}(p^2) = \hat{\Pi}_{zA}^{(\mathrm{scalar})}(p^2)+\hat{\Pi}_{zA}^{(\mathrm{quark})}(p^2) \tag{70}$$

and each of these terms is given as follows:

$$\hat{\Pi}_{zz}^{(\mathrm{scalar})}(p^2) = \frac{m_h^4}{v^2}\cos^2(\alpha-\beta) f_2(p^2,m_h^2,0)$$

$$+\frac{m_H^4}{v^2}\sin^2(\alpha-\beta)f_2(p^2,m_H^2,0)$$
$$+\frac{(m_h^2-m_A^2)^2}{v^2}\sin^2(\alpha-\beta)f_2(p^2,m_h^2,m_A^2)$$
$$+\frac{(m_H^2-m_A^2)^2}{v^2}\cos^2(\alpha-\beta)f_2(p^2,m_H^2,m_A^2),$$

$$(71)$$

$$\hat{\Pi}_{zz}^{(\text{quark})}(p^2)=2N_C\frac{m_t^2}{v^2}f_2(p^2,m_t^2,m_t^2)p^2, \quad (72)$$

$$\begin{aligned}\hat{\Pi}_{zA}^{(\text{scalar})}(p^2) \;=\; & \frac{m_h^2(m_h^2-m_A^2)}{2v^2}\sin(2\alpha-2\beta)f_2(p^2,m_h^2,0)\\ & -\frac{m_H^2(m_H^2-m_A^2)}{2v^2}\sin(2\alpha-2\beta)f_2(p^2,m_H^2,0)\\ & +\frac{m_h^2-m_A^2}{v^2}\sin(\alpha-\beta)\{m_h^2(\frac{\sin^2\beta\cos\alpha}{\cos\beta}+\frac{\cos^2\beta\sin\alpha}{\sin\beta})\\ & +2m_A^2\cos(\alpha-\beta)\}f_2(p^2,m_h^2,m_A^2)\\ & +\frac{m_H^2-m_A^2}{v^2}\cos(\alpha-\beta)\{m_H^2(\frac{\cos^2\beta\cos\alpha}{\sin\beta}-\frac{\sin^2\beta\sin\alpha}{\cos\beta})\\ & -2m_A^2\sin(\alpha-\beta)\}f_2(p^2,m_H^2,m_A^2),\end{aligned}$$

$$(73)$$

$$\hat{\Pi}_{zA}^{(\text{quark})}(p^2)=2N_C\frac{m_t^2}{v^2}\cot\beta f_2(p^2,m_t^2,m_t^2)p^2. \quad (74)$$

Similarly, $\Pi_{ww}(p^2)$ and $\Pi_{wG}(p^2)$ are given in the same form as in Eqs. (69) and (70). $\hat{\Pi}_{ww}^{(\text{scalar})}(p^2)$ and $\hat{\Pi}_{wG}^{(\text{scalar})}(p^2)$ were give explicitly in Ref. [5] and the top quark contributions are

$$\hat{\Pi}_{ww}^{(\text{quark})}(p^2)=-2N_C\frac{m_t^2}{v^2}(m_t^2-p^2)f_2(p^2,m_t^2,0), \quad (75)$$

$$\hat{\Pi}_{wG}^{(\text{quark})}(p^2)=-2N_C\frac{m_t^2}{v^2}\cot\beta(m_t^2-p^2)f_2(p^2,m_t^2,0). \quad (76)$$

Appendix D

The other two-point functions $\Pi_{HH}(p^2)$ and $\Pi_{hH}(p^2)$ were given in Appendices C and D in Ref. [5] except for the quark contributions,

$$\begin{aligned}
\Pi_{HH}^{(\text{quark})} &= -2N_C \frac{m_t^2}{v^2}\frac{\cos^2\alpha}{\sin^2\beta}\left\{2f_1(m_t^2)+(4m_t^2-p^2)f_2(p^2,m_t^2,m_t^2)\right\} \\
&\quad +\frac{N_C}{v}\left(\frac{\cos^3\alpha}{\sin\beta}-\frac{\sin^3\alpha}{\cos\beta}\right)T_H^{(\text{quark})}+\frac{N_C}{v}\frac{\sin 2\alpha}{\sin 2\beta}\cos(\alpha-\beta)T_h^{(\text{quark})},
\end{aligned} \tag{77}$$

$$\begin{aligned}
\Pi_{hH}^{(\text{quark})} &= -2N_C \frac{m_t^2}{v^2}\frac{\sin\alpha\cos\alpha}{\sin^2\beta}\left\{2f_1(m_t^2)+(4m_t^2-p^2)f_2(p^2,m_t^2,m_t^2)\right\} \\
&\quad +\frac{N_C}{v}\frac{\sin 2\alpha}{\sin 2\beta}\left\{\cos(\alpha-\beta)T_H^{(\text{quark})}+\sin(\alpha-\beta)T_h^{(\text{quark})}\right\}.
\end{aligned} \tag{78}$$

Here the tad pole effects on the H and h fields due to the top quark are denoted by $T_H^{(\text{quark})}$ and $T_h^{(\text{quark})}$ respectively. These are given by

$$T_H^{(\text{quark})} = 4\frac{m_t^2}{v}\frac{\cos\alpha}{\sin\beta}f_1(m_t^2), \tag{79}$$

$$T_h^{(\text{quark})} = 4\frac{m_t^2}{v}\frac{\sin\alpha}{\sin\beta}f_1(m_t^2). \tag{80}$$

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Table 1.

Yukawa couplings in (65).

Coefficients	Model I	Model II
C_1	$-(m_b/v)\cos\alpha/\sin\beta$	$(m_b/v)\sin\alpha/\cos\beta$
C_2	$-(m_b/v)\sin\alpha/\sin\beta$	$-(m_b/v)\cos\alpha/\cos\beta$
C_3	$-m_b/v$	$-m_b/v$
C_4	$-(m_b/v)\cot\beta$	$(m_b/v)\tan\beta$
D_1	$-(m_t/v)\cos\alpha/\sin\beta$	$-(m_t/v)\cos\alpha/\sin\beta$
D_2	$-(m_t/v)\sin\alpha/\sin\beta$	$-(m_t/v)\sin\alpha/\sin\beta$
D_3	m_t/v	m_t/v
D_4	$(m_t/v)\cot\beta$	$(m_t/v)\cot\beta$
E_1	$-m_b/\sqrt{2}v$	$-m_b/\sqrt{2}v$
E_2	$-(m_b/\sqrt{2}v)\cot\beta$	$(m_b/\sqrt{2}v)\tan\beta$
F_1	$m_t/\sqrt{2}v$	$m_t/\sqrt{2}v$
F_2	$(m_t/\sqrt{2}v)\cot\beta$	$(m_t/\sqrt{2}v)\cot\beta$

Table 2.

Combinations of internal particles (X, Y) running in Fig. 1(a) and X in Fig. 1(b) contributing to Π_{zz} and Π_{zA} .

Propagator	type	Internal particle species
Π_{zz}	Fig. 1(a)	(H, z), (H, A), (h, z), (h, A), $(t, \bar{t})$
	Fig. 1(b)	G, A, h, H
Π_{zA}	Fig. 1(a)	(H, z), (H, A), (h, z), (h, A), $(t, \bar{t})$
	Fig. 1(b)	G, A, h, H

Table 3.

Combinations of internal particles (X, Y; Z) in Fig. 2(a) and (X, Y) in Figs. 2(b) and 2(c) for the vertices $\Gamma^{(i)}_{Hzz}$ $(i = 1, 2, 3)$ and $\Gamma^{(\mathrm{quark})}_{Hzz}$.

Vertex	type	Internal particle species
$\Gamma^{(1)}_{Hzz}$	Fig. 2(a)	$(z, z; h)$, $(z, z; H)$, $(A, A; h)$, $(A, A; H)$,
		$(A, z; H)$, $(A, z; h)$, $(H, H; z)$, $(H, H; A)$,
		$(H, h; z)$, $(H, h; A)$, $(h, h; z)$, $(h, h; A)$,
$\Gamma^{(2)}_{Hzz}$	Fig. 2(b)	(w, w), (z, z), (G, G), (A, A), (A, z),
		(G, w), (H, H), (H, h), (h, h)
$\Gamma^{(3)}_{Hzz}$	Fig. 2(c)	(z, h), (z, H), (A, h), (A, H),
$\Gamma^{(\mathrm{quark})}_{Hzz}$	Fig. 2(a)	$(t, \bar{t}; b)$, $(b, \bar{b}; t)$

Figure Captions

Fig. 1 Three types of Feynman diagrams contributing to the two-point functions Π_{ij}, where i and j are either of H, h, G or A. Fig. 1 (c) denotes the tad pole contributions. Internal particles (X, Y) in (a) and X in (b) are listed in Table 2.

Fig. 2 Three types of Feynman diagrams contributing to the Hzz vertex; (a) $\Gamma^{(1)}_{Hzz}$ and $\Gamma^{(\mathrm{quark})}_{Hzz}$, (b) $\Gamma^{(2)}_{Hzz}$, and (c) $\Gamma^{(3)}_{Hzz}$. Internal particles $(X, Y; Z)$ in (a) and (X, Y) in (b) and (c) are given in Table 3.

Fig. 3 The decay width $\Gamma(H \to Z_L Z_L)$ ((a), (b) and (c)) and $\Gamma(H \to W_L^+ W_L^-)$ ((A), (B) and (C)) as a function of m_G. The choice of m_A is varied as 400 GeV ((a) and (A)), 600 GeV ((b) and (B)) and 800 GeV ((c) and (C)). Mixing angles are determined by $\tan\beta = 2$, and $\sin^2(\alpha - \beta) = 1$, and the neutral CP-even Higgs boson masses are $m_H = 300$ GeV and $m_h = 400$ GeV.

Fig. 4 The decay width $\Gamma(H \to Z_L Z_L)$ ((a), (b) and (c)) and $\Gamma(H \to W_L^+ W_L^-)$ ((A), (B) and (C)) as a function of m_G. The choice of m_A is varied as 400 GeV ((a) and (A)), 600 GeV ((b) and (B)) and 800 GeV ((c) and (C)). Mixing angles are determined by $\tan\beta = 10$, and $\sin^2(\alpha - \beta) = 0.5$, and the neutral CP-even Higgs boson masses are $m_H = 300$ GeV and $m_h = 400$ GeV.

Fig. 5 The bird-eye's view of the decay width $\Gamma(H \to Z_L Z_L)$ as a function of m_G and m_A. The mixing parameters are determined by $\tan\beta = 2$, and $\sin^2(\alpha - \beta) = 1$, and the neutral CP-even Higgs boson masses are $m_H = 300$ GeV and $m_h = 400$ GeV.

Fig. 6 The bird-eye's view of the decay width $\Gamma(H \to Z_L Z_L)$ as a function of m_G and m_A. The mixing parameters are determined by $\tan\beta = 10$, and

$\sin^2(\alpha - \beta) = 0.5$, and the neutral CP-even Higgs boson masses are $m_H = 300$ GeV and $m_h = 400$ GeV.

Fig. 7 The bird-eye's view of the decay width $\Gamma(H \to W_L^+ W_L^-)$ as a function of m_G and m_A. The mixing parameters are determined by $\tan\beta = 2$, and $\sin^2(\alpha - \beta) = 1$, and the neutral CP-even Higgs boson masses are $m_H = 300$ GeV and $m_h = 400$ GeV.

Fig. 8 The bird-eye's view of the decay width $\Gamma(H \to W_L^+ W_L^-)$ as a function of m_G and m_A. The mixing parameters are determined by $\tan\beta = 10$, and $\sin^2(\alpha - \beta) = 0.5$, and the neutral CP-even Higgs boson masses are $m_H = 300$ GeV and $m_h = 400$ GeV.

Fig. 9 The mixing angle α dependence of the decay width $\Gamma(H \to Z_L Z_L)$. The mixing angle β is determined by (a) $\tan\beta = 2$, (b) $\tan\beta = 5$ and (c) $\tan\beta = 10$, respectively. The Higgs boson masses are $m_H = 300$ GeV, $m_h = 400$ GeV, $m_G = 500$ GeV and $m_A = 600$ GeV.